

## II. cosmic rays

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## II. COSMIC RAYS

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### § 1. INTRODUCTION

THE last few years have been a time of rapid and decisive progress in our knowledge of cosmic rays. This has been due to the development of the experimental technique as well as to the progress made in the theoretical understanding of the cosmic-ray phenomena.

To mention only some of the important experiments which have been made recently: It has been possible to measure the intensity of cosmic rays at very great heights during balloon flights extending practically up to the top of the atmosphere. Together with the measurements in deep lakes and mines, we have now a full account of the cosmic-ray intensity ranging from nearly the outside of the earth down to 800 m. below sea level.

One of the vital questions in the problem of cosmic rays was that of the energy loss of cosmic-ray particles in traversing matter. This question could be brought to an experimental decision by a refinement of the technique used in measuring the curvature of tracks in a cloud chamber. It is now possible to measure the energies of cosmic-ray particles, and, therefore, also the energy loss, with great accuracy for energies up to several thousand million ev.

On the other hand, the situation could be very much clarified from the theoretical side. In fact an application of the ordinary (relativistic) quantum theory to very fast electrons has revealed the fact that in the region of cosmic-ray energies the behaviour of electrons and light quanta is entirely different from the behaviour at energies obtained from radioactive or artificial sources. Indeed, the theory leads exactly to the type of phenomena observed in cosmic rays. One of the most striking facts—the formation of showers—has found a simple and natural explanation by the laws of ordinary quantum theory. Further investigations have then shown that all the phenomena connected with the so-called soft component of cosmic radiation can be understood more or less quantitatively on the basis of this theory and that the laws of quantum theory are valid for electrons up to the highest energies known at present.

There are, however, facts which could not be explained in this way. The extremely great penetrating power of some of the cosmic-ray particles (the hard component) is in direct contradiction to the quantum theory if the latter is assumed to be valid for electrons. Since this was verified both by direct experiments and by the success with which the theory was applied to the soft component, the only possible conclusion was, that the penetrating particles were not electrons. More recent experiments have in fact shown that we have to deal here with a new type of particle having a mass between that of an electron and proton, e.g. of the order of 100 to 200 electron masses, called the "heavy electron".

The heavy electron discovered in cosmic rays has already proved to be a particle of fundamental importance. It is intimately connected with the short range forces acting between a neutron and proton and also with the anomalous magnetic moments of these particles. It bears the same relation to the nuclear forces as light quanta bear to the electric forces between two charged particles. We shall discuss the connexion between the nuclear properties and the heavy electron in the last chapter of this report. Although it was possible to build up a qualitative theory of the properties of the nuclear particles based on the heavy electron, this theory is still unsatisfactory in many respects and our knowledge about the heavy electron is still very limited. On the other hand, there can be no doubt about the fact that this particle holds one of the key positions for the future nuclear physics and quantum electrodynamics, and that further investigations of its properties will contribute substantially to the final solution of these problems. We believe it will mainly be for experiment to take the lead at present.

## § 2. FUNDAMENTAL EXPERIMENTAL RESULTS

*Soft and hard component.* In the earlier experiments the intensity of cosmic radiation was measured by an ionization chamber. This method is now usually replaced by an arrangement of two (or more) Geiger-Müller counters placed in a vertical plane and simultaneous discharges of the counters are recorded. In this way the number of single charged particles coming from the vertical direction (or any other given direction) is counted, irrespective of the nature and energy of these particles. It will be seen, however, that in cosmic radiation also neutral particles occur (light quanta, perhaps neutrons and other massive neutral particles) which are not recorded by this method.

In order to gain some knowledge about the properties of the cosmic-ray particles we consider their penetrating power in matter. This can be measured in the following way: A block of some absorbing material (lead, aluminium) with varying thickness is placed between the counters, and the number of particles passing through the block is measured as a function of the thickness.

Auger, Ehrenfest and Leprince-Ringuet<sup>(5)</sup> were the first who measured in this way the penetrating power of cosmic-ray particles in various materials and at different heights above sea level. The result is given in figure 1 for lead at a height of 3500 m. We see that the number of cosmic-ray particles decreases very rapidly within the first 5 cm. of lead. For larger thicknesses it decreases only very slowly. A certain fraction (about 50 per cent) of the particles is therefore absorbed in 5 cm. of lead, whereas the rest have a much larger penetrating power, up to 1 m. of lead and more. At sea level the absorbable part is only 20 per cent and decreases further below sea level<sup>(41)</sup>. It was from this result that Auger, Ehrenfest and Leprince-Ringuet first concluded that the cosmic-ray particles can be divided into two groups: a soft group (at sea level 20 per cent) with a penetrating power of not more than 5 cm. of lead, and a hard group with a much larger penetrating power. The distinction between these two groups will prove to be fundamental for the discussion

of all cosmic-ray phenomena and indeed we shall see below that the particles constituting these two groups are different in nature: the soft particles are ordinary electrons, whereas the hard particles are heavy electrons with a rest mass of about 100 or 200 electron masses.

*Intensity curve in the atmosphere.* The same counter arrangement has served for intensity measurements covering the whole range of the atmosphere and below the earth's surface. Regener and Pfitzer<sup>(54,55)</sup> have succeeded in sending an apparatus recording the countings automatically in a balloon up to nearly the top of the atmosphere. It was found that the intensity increases rapidly with increasing height above sea level and reaches a maximum very near to the top of the atmosphere, at a pressure of 8 cm. Hg. The maximum intensity is 40 times larger than that at sea level. For still larger heights the intensity decreases again and reaches

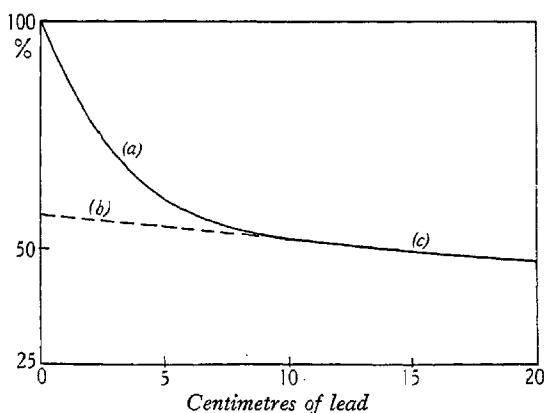


Figure 1. Absorption of vertical cosmic-ray particles (measured on Jungfraujoch). Soft (a) and hard (b) components.

a certain finite value at the top. The intensity at the top is about one third, certainly less than one half, the maximum intensity.

A certain fraction of the total intensity is due to the hard and soft groups respectively. Measurements (with a lead block of 5–10 cm. between the counters) have so far been carried out only to heights of about 4000 m. The intensity of the soft group increases much more rapidly than that of the hard group. At Jungfraujoch the intensities are about equal (at sea level hard : soft = 4 : 1). In the high layers of the atmosphere the intensity of the hard group is certainly very small compared with that of the soft one.\* By extrapolation we find that the ratio hard : soft is only about 1 : 10 at the maximum.

In figure 2 we have plotted the intensity of the soft group only, obtained by subtracting the intensity of the hard group from the total intensity. The fact that this intensity curve shows a maximum near the top of the atmosphere leads to the conclusion that the primary radiation—whatever its nature may be—produces a large number of secondary particles in entering the atmosphere. The secondaries

\* By ionization measurements this was also shown by Compton and Stephenson<sup>(26)</sup>.

belong mainly to the soft group and, indeed, the bulk of soft particles are those secondaries produced in the atmosphere.

The intensity of the vertical radiation has also been measured below the surface of the earth. This has been done with a counter arrangement by Ehmert<sup>(27)</sup> in Lake Constance, and by Barnóthy and Forró<sup>(7)</sup> in deep mines. Thus we have now a full account of the intensity curve ranging from the top of the atmosphere down to a depth of nearly 1000 m. water equivalent below the top. The intensity decreases steadily (as Ehmert has shown very nearly with the power  $h^{1.87}$  of the depth  $h$ ). At a depth of 100 m. water the intensity is 2 per cent of the intensity at sea level, at a depth of 800 m. water only  $\frac{1}{2}$  per cent.

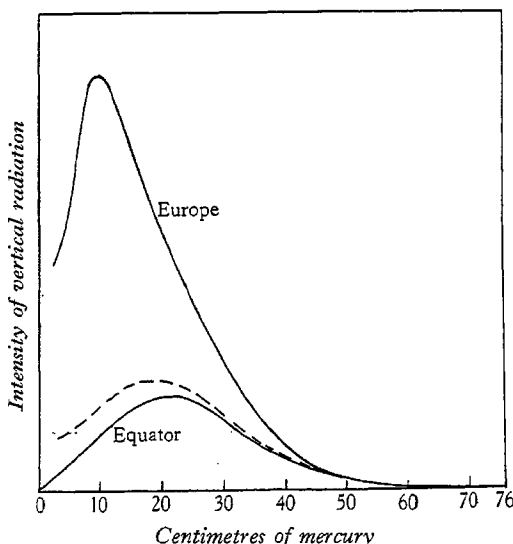


Figure 2. Vertical intensity in the atmosphere in Europe (experiments by Pfozter) and at the equator. The full line is theoretical, the dotted experimental. (Bowen, Millikan, Neher.)

*Latitude effect.* It was first found by Clay<sup>(25)</sup> that the intensity of the cosmic radiation varies with the geographical latitude. The intensity is smaller in the equatorial belt than in Europe. At sea level this variation amounts to only about 20 per cent of the total intensity between European latitudes and the equator. Extensive measurements of the latitude effect covering the whole range of the atmosphere have been carried out by Bowen, Millikan and Neher<sup>(19)</sup>. The latitude effect is very much larger in the high atmosphere. In figure 2 we have also plotted the intensity at the equator (dotted line). We see that at a pressure of 10 cm. Hg the intensity is only a small fraction of that at  $50^\circ$  latitude. In the high atmosphere the latitude effect is mainly due to the soft component since the hard one is practically negligible there. It is, however, probable that the hard component will also show a certain latitude effect. At sea level both components contribute to the effect.

The effect is caused by the magnetic field of the earth which deflects charged particles coming from the outside of the earth. Although the field is very weak it extends over enormous regions and has, therefore, a considerable influence on

particles with energies even as high as  $10^{10}$  ev.\* From the existence of the latitude effect we can conclude that the majority of all incoming rays are charged particles (from which the particles observed at sea level originate as secondaries). The orbits of a charged particle in the magnetic field of the earth have been studied by Lemaître and Vallarta<sup>(46)</sup>. One of their most important results is the following: charged particles can reach the surface of the atmosphere from the universe only if their energy is larger than a certain critical minimum energy  $E_\lambda$  which depends upon the geographical latitude  $\lambda$  and the direction of their orbit at the earth's surface (or the surface of the atmosphere). For particles reaching the earth in the vertical direction these minimum energies are:

$$\begin{array}{ccccccc} \lambda & & 0 & & 50^\circ & & 90^\circ \\ E_\lambda & & 2 \times 10^{10} & & 3 \times 10^9 & & 0 \text{ ev.} \end{array} \quad \dots\dots(1)$$

Thus the magnetic field of the earth has a blocking effect preventing slow particles from reaching the earth. From (1) we can conclude that the latitude-sensitive part of cosmic radiation originates from primaries with energies between  $3 \times 10^9$  and  $2 \times 10^{10}$  ev. whilst the rest originates either from primaries with energies larger than  $2 \times 10^{10}$  ev. or from a neutral primary radiation. Probably the first alternative is true and there is at present no indication of any neutral primary radiation.

*Energy and mass of cosmic-ray particles.* The energy of cosmic-ray particles can be measured in a cloud chamber placed in a strong magnetic field  $H$ . The tracks show then a curvature  $\rho$  which is given by the formula

$$eH\rho = E \sqrt{1 - \left(\frac{mc^2}{E}\right)^2}, \quad \dots\dots(2)$$

where  $e$  is the charge,  $m$  the mass and  $E$  the relativistic energy of the particle. If  $e$  and  $m$  are known  $E$  can be calculated from the measured curvature  $\rho$ . If  $E$  is large compared with the rest energy  $mc^2$  the curvature depends only on  $E$  and is independent of  $m$ .  $E$  can then be calculated without knowledge of the mass. If the mass is not known another independent measurement is necessary to determine  $E$  and  $m$ .† For this purpose a measurement of the specific ionization produced by the particle in the cloud chamber can serve. The specific ionization of a fast particle in standard air depends on the energy in a way shown in figure 3. For small energies it decreases rapidly with increasing energy and reaches a minimum at a kinetic energy equal to the rest energy  $mc^2$  of the particle. For still larger energies the ionization remains more or less constant or, more precisely, it increases logarithmically with increasing ratio  $E/mc^2$ . For particles with different rest masses the ionization curves are practically the same, they have merely to be shifted along the energy axis so that the minimum lies at a kinetic energy equal to the rest energy  $mc^2$ . The minimum value is the same for all particles with the same charge  $e$ . If, therefore, a cosmic-ray track shows an ionization not appreciably higher than

\* It is even probable that the magnetic field of the sun has an influence on the cosmic radiation (see Janossy<sup>(43)</sup> and Epstein<sup>(29)</sup>).

† The charge of all cosmic-ray particles is  $\pm$  the electronic charge. Not a single case has been discovered so far which would favour a different charge. This is concluded from ionization measurements (see below).

the minimum ionization (we call it the normal ionization) we can be sure that the particle has an energy  $\gg mc^2$ . From a measurement of  $\rho$  we obtain then its kinetic energy and at the same time an upper limit for the mass (because the rest energy is then smaller than  $E$ ). The mass itself cannot then be determined. Only for tracks showing a markedly higher specific ionization than the normal one can  $m$  and  $E$  be determined.

Most of the cosmic-ray particles have an ionization very near to the normal one. Heavily ionizing tracks are very rare. Thus, for most cosmic-ray particles, we can determine directly their energy but not their mass. The latter can only be obtained from indirect arguments based on the penetrating power, etc. This question, together with the determination of the mass from heavily ionizing tracks, will be dealt with in detail below. Here we anticipate only the result. A certain fraction of the cosmic-ray particles are ordinary electrons. They will be seen to

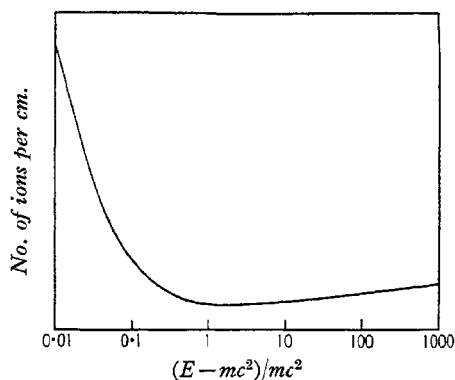


Figure 3. Specific ionization of a fast particle (or specific energy loss by ionization) as a function of the energy.  $mc^2$  is the rest energy of the colliding particle.

constitute the soft component. The hard component consists of particles with a rest mass between 100 and 200 electron masses. Protons (and  $\alpha$ -particles) do not seem to occur in cosmic rays in appreciable numbers except as a result of nuclear disintegrations produced by cosmic-ray particles.

The most accurate and most extended energy measurements have been carried out by Blackett and Brode<sup>(13,14)</sup> (cf. ref. (1)). They were able to measure energies up to  $10^{10}$  ev. According to their results the frequency of occurrence  $N(E) dE$  of particles with energy between  $E$  and  $E + dE$  follows approximately an  $E^{-2.5}$  law for energies greater than  $5 \times 10^8$  or  $10^{10}$  ev.:

$$N(E) dE = \text{const.} \frac{dE}{E^{2.5}}. \quad \text{.....(3)}$$

It will be seen below that this energy spectrum refers mainly to the hard component. For energies less than  $5 \times 10^8$  ev. it is very difficult to determine the energy spectrum because those particles are frequently produced as secondaries in the walls of the chamber or in the surrounding materials (magnet), a fact by which the primary energy spectrum is, of course, very much falsified.

Further experiments, in particular on the energy loss of cosmic-ray particles in traversing matter, will be discussed below.

*Showers.* We have seen in the preceding section 2 that the primary cosmic-ray particles are capable of producing a large number of secondaries in passing through the atmosphere. The same is the case if a cosmic-ray particle passes through a sheet of any heavy material (lead). It happens then that a single particle creates a whole bundle of secondaries. This phenomenon is called a *shower* and was first discovered by Blackett and Occhialini<sup>(12)</sup> on direct Wilson chamber photographs. These showers were produced in the metal walls of the chamber itself. A closer study of the phenomenon shows that showers are also frequently produced by non-ionizing rays (light quanta).

The shower phenomenon can be studied more quantitatively by means of Geiger-Müller counters arranged, for instance, as shown in figure 4*a*. The shower is created in the metal plate and simultaneous discharges (coincidences) of all

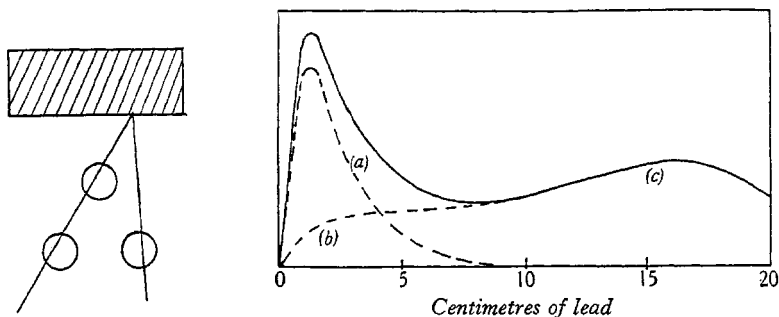


Figure 4*a*.

Figure 4*b*.

Figure 4. Rossi curve. 4*a*, experimental arrangement. 4*b*, number of coincidences as a function of the thickness (schematic). (a), (b), contribution of the soft and hard components respectively.

three counters are recorded. It is necessary that at least two particles (or more if more counters are used) emerge simultaneously from the plate in order to produce such a coincidence. Rossi<sup>(56)</sup> was the first who measured the number of coincidences as a function of the thickness of the shower producing plate. In figure 4*b* we give a typical transition curve for lead<sup>(58, 34)</sup>. It has a maximum at about 15 cm. of lead and then decreases rapidly. After 5 cm. an almost constant value is reached. At 15 to 20 cm. the curve has a flat second maximum. The shape of the Rossi curve shows that the shower production is a complex phenomenon which is certainly due to various types of processes. This was also to be expected since the shower producing radiation consists of two groups with very different penetrating powers.

The curve in figure 4*b* may be considered as composed of two parts (a) and (b). (a) is responsible for the first maximum and decreases to zero within 3 to 5 cm. of lead. This part of the curve is certainly due to showers produced by the soft component, which was seen in § 1 to have a penetrating power of a few cm. of lead. If such a radiation produces secondaries with a penetrating power of the same order of magnitude, i.e. 1 or 2 cm. of lead, we obtain for the number of showers

just a curve of the type (a). At first the number of showers increases with increasing thickness owing to the fact that the probability for producing a shower increases with the amount of material traversed. This, however, is only the case as long as neither the primary nor the secondary radiation is absorbed appreciably. A maximum is reached at a thickness corresponding to the mean penetrating power of these two radiations. For larger thicknesses both the primary and the secondary radiation are absorbed again and the curve decreases rapidly.

The second part (b) of the Rossi curves rises rapidly (within 1 or 2 cm. of lead) to an almost constant value. It is certainly due to a hard shower-producing radiation which yields soft secondaries with a penetrating power of 1 or 2 cm. of lead.\* We shall discuss this part of the curve, together with the second maximum, more in detail below (§ 7).

We have seen above that 80 per cent of all particles at sea level are hard ones. In figure 4 the constant value of part (b) is only about half the maximum value of the soft part (a). Thus we can conclude that the rate of shower production per single primary particle is about 5 to 10 times greater for the soft group than for the hard one. This is also confirmed by the fact found by many observers that the rate of shower production increases with increasing height above sea level much more rapidly than the total intensity of cosmic radiation. It is roughly proportional to the intensity of the soft group which was seen on p. 364 to become more and more preponderant with increasing height. For further details see reference<sup>(38)</sup>.

Cloud chamber photographs of showers show that the great majority of all showers consist of a bundle of 2–20 or more thin tracks with an ionization near to the normal ionization. The energy of the shower particles ranges in most cases between  $10^7$  and  $10^8$  ev. The mass of these particles is, therefore, small and it is certain that most of the shower particles are ordinary positive and negative electrons. The electrons emerge from a centre in more or less the same direction and the angular spread of a shower is—except for very large showers—quite small.

In some cases very large showers with 100–1000 particles have been observed. They are usually called “bursts”. The phenomenon had already been discovered a long time ago by Hoffmann<sup>(42)</sup> in an ionization chamber. The ionization due to cosmic rays was recorded as a function of time. Occasionally a sudden very large increase of ionization (“Stoss”) was observed. If this sudden burst of ionization was due to particles with normal ionization, a very large number of particles (100 to several thousand) must have entered the chamber simultaneously. It is now quite certain that these bursts are identical with the very large showers observed in the cloud chamber, and it has been shown by Bøggild<sup>(18)</sup> and Carmichael<sup>(24)</sup> that the Rossi curve for bursts is very similar to that for small showers, the only difference being that the contribution from the hard component is comparatively larger.

In some cases showers of a type quite different from the bundle showers have been observed<sup>(22)</sup>. In contrast to the latter they contain heavily ionizing tracks (accompanied by thin tracks) and these particles are emitted in all directions from

\* The part (b) of the shower curve can be measured more easily below sea level where the hard component is very much more preponderant<sup>(6)</sup>.

a point. The ionization of the thick tracks shows that these particles are in all probability protons (at least in most cases). It is probable that we have to deal here with nuclear disintegrations produced by cosmic rays. These showers can also be observed directly in a photographic plate exposed for some time to cosmic rays<sup>(17)</sup>. The shower produced in the emulsion of the plate appears as a little star consisting of a number of proton tracks emitted all from a single point (thin tracks are not recorded by this method). We discuss this type of shower in the last section.

### § 3. THEORY OF THE PASSAGE OF FAST ELECTRONS THROUGH MATTER

From the experiments discussed in the preceding section it seems very probable that at least a certain fraction of the cosmic-ray particles are ordinary positive and negative electrons (for instance, the shower particles). For a theoretical understanding of the cosmic-ray phenomena it will thus be the natural way to try to apply the ordinary (relativistic) quantum theory to electrons. In cosmic rays we have, however, to deal with particles having energies 10 to 10,000 times larger than the highest energies available from artificial or radioactive sources and it is by no means a priori certain whether our present quantum theory still holds for those high energies. The question whether or not the quantum theory breaks down for high energies is, of course, one of fundamental importance. It can only be decided by working out in detail the consequences of the theory for electrons and by comparing the results with the experiments. This will be done in the following section. It will be seen that all the important phenomena connected with the soft radiation can be understood on grounds of the quantum theory of the electron and that the latter holds for energies up to the highest known at present.

*Energy loss of fast electrons.* A fast particle passing through matter loses its energy mainly in two ways:

(a) In passing through the electronic shell of an atom energy is transferred to one of the electrons by an inelastic collision. This is the familiar process of ionization by fast particles. The mean energy transferred to the electron is practically independent of the energy of the primary particle. The curve shown in figure 3 giving the ionization of a fast particle stands, therefore, approximately also for the mean energy lost by the primary particle. The energy loss per cm. path of a fast electron with an energy much greater than  $mc^2$  is given by the following formula:

$$\left(-\frac{\partial E}{\partial x}\right)_{\text{coll.}} = NZ2\pi \left(\frac{e^2}{mc^2}\right)^2 mc^2 \log \frac{E^3}{2mc^2 I^2 Z^2}, \quad \dots\dots(4)$$

where  $N$  is the number of atoms per  $\text{cm}^3$ ,  $Z$  the atomic number,  $I$  a certain mean ionization energy, viz.  $I = 13.5$  ev. The specific energy loss is proportional to the number of electrons  $NZ$  per unit volume, e.g. to  $Z$ .

(b) If an electron passes through the electric field of an atom it is deflected, which is equivalent to a certain acceleration of the electron. Thus radiation will always be emitted in such a collision. The process is known as ordinary emission of a continuous x-ray spectrum. For cosmic-ray energies it plays the most important

part for the energy loss of the electron. The theory of this process has been developed by Bethe and Heitler<sup>(8,37)</sup>. We confine ourselves to a brief summary of the results.\*

An electron moving with cosmic-ray energies suffers in general only very small deflections in passing through an atom, but even a small deflection represents, for a very fast particle, a large acceleration and the emission of light becomes, therefore, very probable. The light quantum is emitted in practically the same direction as the electron, e.g. in the forward direction. The theory shows now that there is a large probability for the emission of a light quantum having an energy  $h\nu$  comparable with the energy  $E$  of the primary electron itself. If we express the energy of the light quantum as a fraction of  $E$ ,  $\alpha = h\nu/E$ , the probability of an electron losing the fraction  $\alpha$  of its energy is, for high energies, independent of  $E$  itself. The mean energy lost by the electron in passing through an atom increases, therefore, proportionately to the energy  $E$  of the electron. The frequency distribution of the energy lost is nearly uniform, thus, for instance, three-quarters of the energy is lost in the form of quanta having each an energy larger than  $E/4$ .

The total mean energy loss of an electron by radiation per cm. path is given by the formula (for very high energies)

$$\left(-\frac{\partial E}{\partial x}\right)_{\text{rad.}} = NZ^2 \left(\frac{e^2}{mc^2}\right)^2 \frac{E}{137} \left[2.1 - \frac{4}{3} \log Z\right]. \quad \dots\dots(5)$$

It has, however, to be born in mind that this energy is not lost steadily, but in the form of a few hard quanta with energy  $h\nu$  each comparable with  $E$  itself. The energy lost by a particular electron in traversing a thin plate of matter fluctuates therefore considerably about the mean loss given by equation (5). Furthermore, in contrast to the energy loss by ionization, (5) is proportional to the square of the nuclear charge  $Z$ .

Since  $(\partial E/\partial x)_{\text{rad.}}$  increases  $\sim E$  there will be a certain critical energy  $E_c$  for which  $(\partial E/\partial x)_{\text{coll.}} = (\partial E/\partial x)_{\text{rad.}}$ . For energies  $E > E_c$  the energy loss by radiation will be the only important one, all other modes of energy loss being then negligible. This critical energy depends on  $Z$  and is higher for light elements than for heavy ones.

Comparing (4) with (5) we find easily that  $E_c$  lies at

|                         |                 |                     |                 |
|-------------------------|-----------------|---------------------|-----------------|
| Air and<br>water        | Al              | Pb                  |                 |
| $E_c = 1.3 \times 10^8$ | $6 \times 10^7$ | $1 \times 10^7$ ev. | $\dots\dots(6)$ |

Thus, cosmic-ray electrons will practically lose their energy only by radiation.

*Absorption of light quanta.* The absorption laws of light quanta are very similar to those of fast electrons. At low energies light quanta are absorbed by the photoelectric effect and by Compton scattering. At cosmic-ray energies, the only important mode of absorption is that by the creation of a pair of a positive and negative electron. There is again a critical energy  $E_c$  so that for  $E > E_c$  all other processes can be neglected whilst for  $E < E_c$  the Compton scattering (and for still smaller energies the photoelectric effect) is more important than pair production.  $E_c$  is

\* Cf. also the article by Collie, *Progress Reports* (1937).

about the same as for electrons, equation (6). The absorption coefficient for hard light quanta due to pair creation is given by the formula (for very high energies)

$$\tau = N \frac{Z^2}{137} \left( \frac{e^2}{mc^2} \right)^2 \left[ 16 \cdot 1 - \frac{28}{27} \log Z \right]. \quad \dots (7)$$

Thus, the absorption coefficient does not depend on the frequency at all and is nearly proportional to  $Z^2$ . (The absorption coefficient due to Compton scattering decreases  $\sim 1/\nu$  and is proportional to  $Z$ .)

The formulae (5) and (7) show that fast electrons and hard light quanta are highly absorbable. Their penetrating power does not, or practically not, increase with increasing energy in contrast to the behaviour at lower energies. In normal air a cosmic-ray electron of  $100 \times 10^6$  ev. has a mean range of only 200 m. A light quantum of the same energy has an absorption coefficient of  $0.2 \times 10^{-4} \text{ cm}^{-1}$  (mean range of 500 m.) In lead these figures are 1.5 cm. and 1  $\text{cm}^{-1}$  respectively.

At first sight it seems that this theory leads to a penetrating power for electrons far too small to account for the observed behaviour of cosmic particles. But this difficulty is only an apparent one and we shall see in § 4 that the theory is fully confirmed by the cosmic-ray facts.

#### § 4. THEORY OF CASCADE SHOWERS

We have seen that a fast electron loses its energy in matter within a very small range. In order to apply the theory to the actual passage of cosmic-ray electrons and light quanta through thick layers we have to consider the way in which the energy is dissipated more in detail. According to § 3 the energy lost by the electron is not divided into a very large number of small quanta (as is the case for the energy lost by ionization) but reappears in the form of a few hard light quanta with energies each comparable with the energy of the primary electron. All these quanta are practically emitted in the forward direction. They carry a large fraction of the energy of the original electron somewhat farther through the layer until they are absorbed (a small fraction of the energy is lost in the form of soft quanta emitted in all directions). The hard light quanta are absorbed by creation of an electron pair, the whole energy of each quantum being divided up between the two pair electrons. Each of these pair electrons will, therefore, have an energy, though of course smaller than, but still comparable with, the primary energy  $E_0$ . At this stage of the process we have instead of one primary electron with energy  $E_0$  a number of positive and negative electrons with energies still comparable with  $E_0$ , although, of course, a certain fraction of  $E_0$  is really lost in the form of soft quanta which cannot produce pairs.

Each of the secondary electrons behaves now in a similar way to the primary electron, e.g. it will emit hard light quanta which produce pairs and so on. This results in a further multiplication of the number of electrons together with a decrease of the energy for each particle. The multiplication takes place as long as the energy of the electrons and light quanta is larger than the critical energy  $E_c$ . As soon as this energy is reached no further multiplication can take place; the electrons then

lose their energy steadily by ionization and the light quanta by Compton scattering. The larger the primary energy is, the more stages are possible and the higher will be the number of electrons produced. Each stage, however, requires a certain thickness of the absorbing layer, which is about equal to the average range which a fast electron travels before its energy decreases by, say, half the primary energy. For thin layers only few stages are possible. In very thick layers the process will result in a complete degradation of energy, all electrons and light quanta being ultimately absorbed. There will be an optimum thickness  $l_m$  at which the number of electrons is a maximum. If we interrupt the process at a thickness near to  $l_m$  and observe the outcoming electrons, say, in a Wilson chamber, we shall find exactly what we have called a *shower*. It is this multiplicative process which is responsible at least for the majority of the showers observed in cosmic radiation.

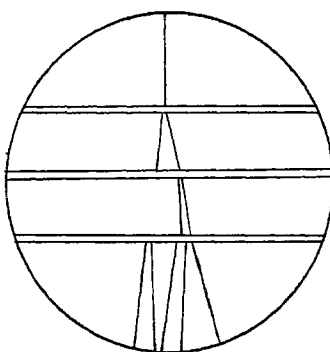


Figure 5. The cascade process for shower production (schematic). The incident particle is multiplied by three successive lead plates.

Recent experiments by Trumpy<sup>(61)</sup> and Fussel<sup>(33)</sup> have shown this directly. A number of very thin lead plates were placed in a cloud chamber some distance apart from each other. It can then be seen how the multiplication process is progressing from plate to plate. We give a schematical picture of such a cascade process in figure 5. The electron on the left-hand side produces a hard light quantum in the second plate which produces then a pair in the third plate.

The cascade shower process has been treated quantitatively independently by Bhabha and Heitler<sup>(9)</sup> and by Carlson and Oppenheimer<sup>(23)</sup>. In the following table 1 we give some of the numerical results. The mean number of electrons (of both signs)  $\bar{Z}$  contained in a shower which is produced by a primary electron with energy  $E$  depends on  $E$ , the material in which the shower is produced and the thickness of the layer. It is convenient to introduce for the thickness a certain unit length depending on the material in question, and representing roughly the distance which an electron travels before it loses its energy by radiation.

The unit thickness is for:

| Normal<br>air | Water | Aluminium | Lead     |          |
|---------------|-------|-----------|----------|----------|
| 280 m.        | 35    | 6.7       | 0.36 cm. | .....(8) |

Furthermore, we introduce instead of the primary energy  $E_0$  the variable

$$y = \log \frac{E}{E_c}, \quad \dots\dots(9)$$

where  $E_c$  is the critical energy defined in equation (6) which also depends on the material.

If these variables are chosen  $\bar{Z}$  is a function of  $l$  and  $y$  only. In table 1 we give the number of shower electrons as a function of  $l$  for various values of  $y$ .  $\bar{Z}$  increases very rapidly with  $y$ . According to this table an electron of  $5 \times 10^8$  ev. would produce in a lead plate of 1.1 cm. ( $l=3, y=4$ ) on the average 6.4 shower electrons. A primary electron with energy  $4 \times 10^9$  ev. would produce a maximum number of more than 50 electrons. Thus we see that even very large showers and Hoffmann bursts can be explained by our theory.

In general, most of the shower electrons have energies near to  $E_c$  (except for very thin plates), but higher and smaller energies also occur.

Table 1. Total mean number of shower electrons\*

| $y \backslash l$ | 1   | 2   | 3    | 5   | 10   | 18   |
|------------------|-----|-----|------|-----|------|------|
| 2                | 1.2 | 1.6 | 1.8  | 1.4 | 0.1  | 0    |
| 4                | 1.9 | 4.0 | 6.4  | 9.6 | 4.1  | 0    |
| 6                | 2.7 | 7.7 | 15.5 | 40  | 54   | 10   |
| 10               | 4.7 | 20  | 57   | 260 | 1800 | 1500 |

From this table we also see how the mean number of shower electrons depends upon the thickness  $l$ .  $\bar{Z}$  starts at a value 1 (primary electron) at a thickness  $l=0$  and rises to a maximum which lies at a value of  $l$  which increases with the primary energy (or  $y$ ). At large thicknesses all electrons are absorbed and  $\bar{Z}$  falls towards zero.

Table 1 gives only the mean number of shower electrons. The actual number of electrons produced fluctuates again largely about the mean value.

From the above considerations it is clear that light quanta are also capable of producing cascade showers in almost exactly the same way as electrons. The first stage of the process is then a pair creation followed by radiation.

## § 5. APPLICATIONS OF THE THEORY

The theory of cascade showers described above has found numerous applications to all sorts of phenomena in cosmic rays. In this report we must confine ourselves to a brief discussion of some of the most important applications.

*The Rossi curve.* If we plot the number of shower electrons  $\bar{Z}$  given in table 1 as a function of the thickness, we obtain a curve which resembles very much the Rossi curve for triple coincidences shown in figure 4b part (a), if the effect due to the hard component, part (b), is subtracted. The theoretical curve for  $\bar{Z}$  depends

\* The numerical calculations have been extended by Arley<sup>(3)</sup>.

upon the primary energy of the electron falling on the top of the shower-producing plate. The position of the maximum shifts to higher thicknesses for higher primary energies. For a rough orientation we might compare the experimental curve with the curve for  $\bar{Z}$  which has the maximum at the same position as the experimental one. This is the case for  $\gamma = \log (E/E_c) = 3$  corresponding for lead to a primary energy (for lead  $E_c = 10^7$  ev.) of  $2 \times 10^8$  ev. We may expect, therefore, that most of the shower-producing electrons have about this energy. This will be seen in § 6 to be the case.

Strictly speaking, the experimental curve cannot be compared directly with the curves representing the mean number of shower electrons. In the first place the experimental curve is the result of primary electrons with all different energies.

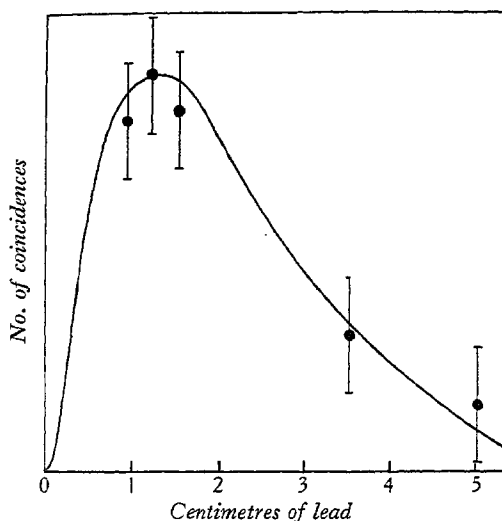


Figure 6. Number of cascade showers containing two, or more than two particles. Theory (full curve) and experimental points by Schwegler. (Experimental arrangement similar to figure 4(a), the hard component is eliminated experimentally.)

To take this into account we should have to know the energy spectrum of the shower-producing electrons and light quanta. This is only roughly known so far; we shall discuss it below. In the second place, what is measured by the experimental arrangement shown in figure 4a is not the mean number of shower electrons, but the number of showers containing a certain minimum number of particles (in figure 4a this number is two). The probability for the production of a shower with, say, more than 2 or 3 particles by a given primary electron can be calculated from the mean number of shower particles, taking into account the fluctuations about this mean number.

Arley<sup>(3)</sup> has calculated in this way the exact shower curve with the energy spectrum discussed in § 6. The result is shown in figure 6 where the (theoretical) probability for the production of a shower of at least two particles is plotted normalized for one incident electron. The experimental points are obtained by Schwegler<sup>(59)</sup>; the scale for the latter is chosen so that the height of the maximum

\* In this experiment the contribution of the hard component is eliminated experimentally.

is the same as for the theoretical curve (the position of the maximum cannot, of course, be chosen arbitrarily). The position of the maximum and the shape of the curve agree very well with the theoretical one.

In a similar way all the other counter experiments can be treated theoretically. For instance, it is possible to calculate the soft part of the absorption curve<sup>(39,3)</sup>, figure 1. On the whole the agreement between theory and experiment is very good, but it is always the soft part of the phenomena which is accounted for by the cascade theory, whilst the hard part remains unexplained. The conclusion is, therefore, that the soft radiation consists of electrons and light quanta which behave in full accordance with the quantum theory.

For Hoffmann bursts Euler<sup>(30)</sup> and Montgomery and Montgomery<sup>(49)</sup> have discussed the transition curve and have found that the cascade theory can well account for a certain fraction of the bursts (50 per cent in lead of 5 cm. thickness). For those very large showers primary energies of  $10^{10}$  to  $10^{11}$  ev. are necessary and these considerations show, therefore, that the quantum theory is still valid at these energies. On the other hand, the contribution of the penetrating component is more preponderant for bursts than for small showers.

*The intensity curve in the atmosphere.* The intensity curve shown in figure 2 has a shape very similar to an ordinary Rossi curve in lead. It is therefore likely that it can be explained in a similar way. The maximum is due to the fact that the rays coming from outside the earth produce numerous secondaries in the upper layers of the atmosphere which are then absorbed again at increasing thickness. In order to compare this curve with our theory we must remember that the energies of the incoming particles are restricted by the magnetic field of the earth. Assuming that the incoming rays are ordinary electrons, we take from equation (1) that all these electrons have energies  $\geq 3 \times 10^9$  ev. at a latitude of  $50^\circ$ . Presumably, the bulk of these particles will have energies just above this minimum energy. Since for air the critical energy  $E_c$  is  $15 \times 10^7$  ev. the  $\gamma$  value for the primary electrons is  $\log(3 \times 10^9 / 15 \times 10^7) = 3$ . According to the results given in table 1, each primary electron produces a total maximum of electrons (including the primary) of 2.5 electrons. The maximum lies at a thickness of  $l = 3$ , i.e. in air 8 cm. Hg, which is exactly what is observed. Also the height of the maximum (about three particles per one primary particle) is in good agreement with the theory although an exact comparison is hardly possible because the number of incident particles is known only approximately from experiments.

The good agreement leaves no doubt about the fact that most of the incident rays are ordinary electrons and that the cascade theory can account for the soft part of the intensity curve, figure 2.

If we consider the intensity curve down to lower heights we find still a considerable number of electrons (quite apart from the hard radiation). If we were to assume only primaries with an energy of  $3 \times 10^9$  ev. the theory would not yield enough electrons at greater depths below the top of the atmosphere. These electrons must be due to primaries with energies above  $3 \times 10^9$  ev., which, though few in number, produce many secondaries at great thicknesses. From the observed intensity in

the atmosphere it is then possible to calculate the energy distribution of the primaries. Let  $F(E) dE$  be the number of primary electrons in the energy interval  $dE$  and  $Z(l, E)$  the total number of particles produced by one primary particle with energy  $E$  at a thickness  $l$ , as given in table 1, the observed intensity  $T(l)$  is

$$T(l) = \int_{3 \times 10^9}^{\infty} F(E) Z(l, E) dE. \quad \dots\dots(10)$$

From this equation  $F(E)$  has been calculated by Heitler<sup>(38)</sup> and Nordheim<sup>(52)</sup> and it has been found that the primary energy distribution is approximately given by the law

$$F(E) = \frac{\text{const}}{E^{2.5}} \quad (\text{for } E > 3 \times 10^9 \text{ ev.}). \quad \dots\dots(11)$$

An experimental test for this law is provided by the latitude effect. In § 2 we have already seen that at the equator all particles which have energies  $\leq 2 \times 10^{10}$  ev. are prevented from reaching the earth. Knowing the primary energy spectrum we are able to predict the intensity curve at the equator. We simply have to cut off the effects of all particles with energy  $< 2 \times 10^{10}$  ev., thus

$$T_{\text{eqn.}}(l) = \int_{2 \times 10^{10} \text{ ev.}}^{\infty} F(E) Z(l, E) dE. \quad \dots\dots(12)$$

The calculated curve  $T_{\text{eqn.}}(l)$  is also plotted in figure 2 (full drawn). The experiments (dotted curve) have been carried out by Bowen, Millikan and Neher<sup>(19)</sup>, and we see that their results are in perfect agreement with our predictions.

Finally, we mention another consequence of the primary energy spectrum equation (11). By the cascade theory and the primary energy spectrum it is not only possible to compute the total number of electrons at each point of the atmosphere, but also their energy spectrum. At sufficiently large thicknesses the result is extremely simple. A primary energy spectrum of the type equation (11) produces at large thicknesses secondaries with energies following the same energy spectrum but for all energies larger than the critical energy  $E_c$ . Thus in air the energy spectrum of the electrons is given by

$$f(E) dE = \frac{\text{const}}{E^{2.5}} dE, \quad E > 1.5 \times 10^8 \text{ ev.} \quad \dots\dots(13)$$

For  $E < 1.5 \times 10^8$  ev.  $f(E)$  is approximately constant. This law is valid for all heights less than about 5000 m. above sea level.

The energy spectrum (13) cannot be compared directly with the measurements (§ 2) because the measured energy spectrum refers to the total radiation (hard + soft), whereas our calculated energy spectrum is that of the soft component only. It is, however, satisfactory that the two components have, at sea level, similar energy distributions. An indirect experimental evidence for the energy spectrum of the soft component is provided by the distribution in size of large showers and bursts<sup>(16, 49)</sup>. This has been discussed by Blackett<sup>(16)</sup> and it was found that the energy distribution, equation (13), was in agreement with the experiments. We discuss the energy spectrum again in § 6.

There are a number of other details connected with the soft component which can be explained satisfactorily on the basis of the cascade theory and its applications to the phenomena in the atmosphere, but for these questions we must refer the reader to the original papers.

#### § 6. DIRECT ENERGY LOSS MEASUREMENTS AND THE NATURE OF THE HARD COMPONENT

*The energy loss curve.* As we have seen in the preceding sections a number of cosmic-ray phenomena have found a qualitative and quantitative explanation by the cascade shower process which is a direct consequence of the quantum theory applied to electrons and light quanta of very high energies. This theory, however, leaves no room for an explanation of the hard component. According to the theory an electron can make its effects felt (by producing secondaries) through layers of at most 5 or 10 cm. of lead, 10 m. of water or 10 km. of normal air. The penetrating cosmic rays are found to pass through layers of 1 m. of lead and more and there are still particles found in mines at a depth of 800 m. below sea level. It is clear that these penetrating particles cannot be electrons if for the latter the cascade theory is adopted for all energies. It was in fact from the success of this cascade theory that several authors<sup>(38, 52, 50)</sup> first came to the conclusion that the penetrating particles are not electrons.

On the other hand, it can also be shown that the penetrating particles cannot be of protonic mass. The ionization of most of these particles is the normal ionization (minimum in figure 3) which shows that their kinetic energy is higher than the rest energy. Deflection experiments in a magnetic field show that these particles have energies down to  $2 \times 10^8$  ev. Since the rest energy of a proton is  $Mc^2 = 10^9$  ev. it is impossible that the penetrating particles have protonic mass. Their rest energy cannot be larger than about  $2 \times 10^8$  ev. corresponding to 400 electron masses. The only possible assumption seems, therefore, to be that the penetrating particles are neither electrons nor protons but represent a hitherto unknown type of particle.

The argument leading to this decisive conclusion rests on the validity of the radiation formulae (5, 7), for the energy loss of electrons up to extremely high energies. The evidence for the validity of the radiation formulae was collected from the phenomena connected with the soft radiation. Although this evidence is very strong it is only indirect, and it is therefore desirable to find a direct experimental proof for the large energy loss of electrons predicted by the theory on the one hand, and for the different behaviour of the penetrating particles on the other hand.

The specific energy loss of a cosmic-ray particle can be measured in a Wilson chamber by placing a metal plate (for instance, 1 cm. lead) across the chamber. In a magnetic field the tracks traversing the plate show a larger curvature below the plate because the particles have lost energy in the plate. From the difference of the curvatures below and above the plate the energy lost by the particle is measured.

The first measurements in this direction were made by Anderson and Neddermeyer<sup>(2)</sup>, while the most accurate and most extended ones are those by Blackett

and Wilson<sup>(15 16,64)</sup>; ref. (7). We give their results schematically in figure 7, where the mean specific energy loss per cm. of lead is plotted as a function of the primary energy. The theory would give for electrons a straight line  $-(1/E) \partial E / \partial x = 2$  (dotted line). The full drawn curve represents the mean value of the experimental energy loss for each energy. As mentioned in § 4 the actual energy lost in each particular case fluctuates largely about this mean value. Up to an energy of  $2 \times 10^8$  ev. the mean energy loss is in exact agreement with the theory. For higher energies the curve drops down rapidly and reaches a value smaller by an order of magnitude than the theoretical loss of an electron. This is not surprising because we already know that there are particles having a much greater penetrating power than electrons would have according to the theory. We now see that these penetrating particles are mainly those which have energies  $> 2 \times 10^8$  ev. whereas the soft particles are mainly those with lower energies. It would, however, be surprising

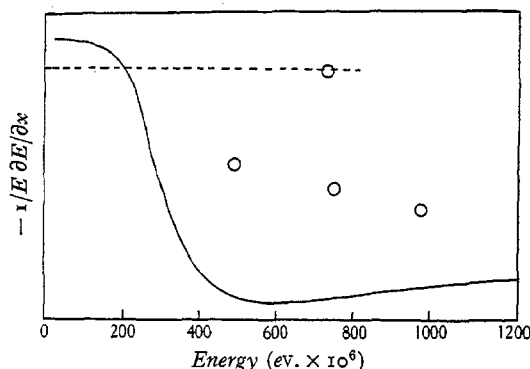


Figure 7. Mean specific energy loss of cosmic-ray particles. Experiments by Blackett and Wilson. The dotted line corresponds to the theoretical value for electrons. In the energy region  $2 \times 10^8$  ev. the graph shows the existence of two groups of particles: a hard group with a small energy loss represented by the full line and a very small number of soft particles (circles) having the energy loss expected for electrons. For low energies ( $2 \times 10^8$  ev.) all particles are electrons obeying the theory.

if these two groups were distinguished so sharply by their energy and the question now arises whether there are also soft particles with energies  $> 2 \times 10^8$  ev. or hard ones with energy less than  $2 \times 10^8$  ev. This is in fact the case.

In the region  $E > 2 \times 10^8$  there have been found a small number of particles having a much larger energy loss than most of the other particles of the same energy. We have marked these particles by a small circle in figure 7. Their energy loss is in agreement with that expected for electrons. Moreover, these particles are almost always associated with showers as is to be expected for electrons of such high energy according to the cascade theory. The number of these highly absorbable particles is comparatively small, not more than 1 or 2 per cent of all particles.

These experiments show directly that in the energy region  $E > 2 \times 10^8$  ev. there are two groups of particles which behave differently and which must therefore be of a different nature. There is a small number of highly absorbable particles which behave like electrons according to the quantum theory and which are certainly

electrons. On the other hand, the bulk of particles with  $E > 2 \times 10^8$  ev. have a small energy loss. These particles cannot therefore be electrons. We discuss their nature below.

The existence of these few tracks with large energy loss provides also a conclusive proof of the fact that for electrons the quantum theory is valid even for very high energies, and there is no reason to believe that there is any limit to the validity of the theory at all. This we had already concluded from the success of the cascade theory applied to large showers.

In the region  $E < 2 \times 10^8$  ev. practically all particles are electrons. In Blackett and Wilson's experiments no particles have been found with considerably greater penetrating power than electrons and energy  $< 2 \times 10^8$  ev.\* Since, however, the individual energy loss fluctuates largely about the mean value it might be that some of the particles with comparatively small energy loss are in fact penetrating particles.

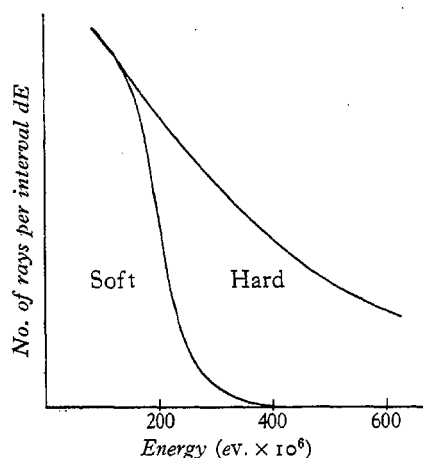


Figure 8. Energy spectra of the soft and hard components (schematic) according to Blackett.

In any case their number is small compared with the number of electrons and, furthermore, they must have lost much more energy than their fellow particles lose in the region of higher energy  $> 2 \times 10^8$  ev. Thus it seems that the energy loss of the penetrating particles increases when their energy passes below  $2 \times 10^8$  ev.

We can now give a schematic picture of the energy distribution of the soft and hard particles separately, figure 8. Most of the hard particles have energies greater than  $2 \times 10^8$  ev. whereas most of the electrons have energies less than  $2 \times 10^8$  ev.

The exact forms of the energy spectra of the two components are only partly known at present, and only for the high energy region. The theory shows that the soft component has an energy spectrum given by equation (13) for  $E > 2 \times 10^8$  ev. The energy measurements (§ 2) show that a similar energy spectrum holds for the total radiation and therefore also for the hard component for  $E > 5 \times 10^8$  ev. The energy spectra for the hard component for  $E < 5 \times 10^8$  ev. and for the soft one for  $E < 2 \times 10^8$  ev. are not known at present.

\* Other authors have found a few heavy electrons with energy less than  $2 \times 10^8$ ; see below.

*The nature of the penetrating particles.* Having seen from direct as well as from indirect evidence that the penetrating particles are neither electrons nor protons, the question arises what the nature of these particles may be. The first question is why these particles lose so little energy. The energy loss of a particle in a given material depends, according to § 3, only on two parameters, the charge  $e$  and the mass  $m$ . The energy loss by ionization depends only on  $e$ ; in formula (4)  $m$  outside the logarithm is the mass of the extranuclear electrons in an atom and is not the mass of the colliding particle in question. The ionization of the penetrating particles is found to be the same as for electrons and  $e$  must therefore be the elementary charge. The energy loss by radiation equation (5) depends also upon the mass  $m$  of the colliding particles and is proportional to  $1/m^2$ . The only way to cut down the large energy loss by radiation is therefore to assume a mass considerably larger than the electronic mass; this suggestion was first made by Neddermeyer and Anderson<sup>(50)</sup>. An upper limit for the mass is given by the fact that for energies greater than  $2 \times 10^8$  ev. the ionization is the minimum one. Thus the mass cannot be larger than  $2 \times 10^8$  ev./ $c^2$ , i.e. 400 electron masses.

According to what we have seen in § 2, the mass of a particle can only be measured if tracks are found showing a larger specific ionization than the minimum one. Those tracks can only be found at energies less than  $2 \times 10^8$  ev. As we have seen above, penetrating particles with such a low energy are extremely rare. In spite of this difficulty several authors have succeeded in finding some of those tracks.\* The method of determining the mass has been discussed in § 2. We shall not give the details here. Most of the tracks measured are those by Williams and Pickup<sup>(63)</sup>. We give their results in table 2.

Table 2

| Momentum (unit<br>$mc$ , $m$ = electron<br>mass) | Mass (unit<br>electron mass) |
|--------------------------------------------------|------------------------------|
| 69                                               | $220 \pm 50$                 |
| 110                                              | 430                          |
| 95                                               | $190 \pm 60$                 |
| 73                                               | $160 \pm 30$                 |

Thus we see that all particles, with one exception, have the same mass within the experimental errors and this mass is about 100 or 200 electron masses. The new particle has been called by several authors the heavy electron.

The heavy electron is thus the particle constituting the penetrating cosmic radiation. We discuss some of its properties and applications in the following section.

\* The first track of this kind with the right interpretation was found by Street and Stevenson<sup>(60)</sup>. For other tracks cf. (57, 51, 28).

## § 7. THE HEAVY ELECTRON AND THE NUCLEAR PROPERTIES

*Yukawa's theory.* The discovery of the heavy electron has already proved to be one of fundamental importance. It seems now quite certain that the heavy electron holds the key position for an understanding of the elementary properties of the nuclear particles proton and neutron. It is well known that between a proton and neutron (and also between two protons and probably between two neutrons) strong forces act with a range of only  $3 \times 10^{-13}$  cm. These forces cannot be of any electromagnetic nature. Furthermore, both particles have an anomalous magnetic moment which, expressed in units of the Bohr nuclear magneton, have the values  $\mu_P = 2.46$ ;  $\mu_N = -1.80$ . These magnetic moments have found no explanation in any of the existent theories.

In a very ingenious paper published in 1935, Yukawa<sup>(65, 66)</sup> proposed a theory of the nuclear forces which was based on the following ideas: Realizing that the electromagnetic forces are always long range forces, he introduced a new type of field which was assumed to satisfy a differential equation leading automatically to short range forces. Whilst the electrostatic potential satisfies Poisson's equation  $\nabla^2\phi = 0$  which, for an electron for instance, has the solution  $e/r$ , Yukawa's potential is assumed to obey the differential equation (in the static case)

$$\nabla^2\phi + \lambda^2\phi = 0 \quad \dots\dots(14)$$

with the solution 
$$\phi = \frac{g}{r} e^{-\lambda r}. \quad \dots\dots(15)$$

$\lambda$  and  $g$  are new universal constants. Equation (15) leads to forces with a range  $1/\lambda$ , and in order to account for the range actually observed we have to choose

$$1/\lambda = 3 \times 10^{-13} \text{ cm.} \quad \dots\dots(16)$$

The constant  $g$  corresponds to the electric charge of a particle in the electromagnetic case. In our case  $g$  is regarded as a new type of charge which determines the strength of the new nuclear field  $\phi$  surrounding a neutron or proton. Both these particles are assumed to have the same  $g$  charge.  $g$  can be determined from the binding energy of the deuteron. At a distance  $1/\lambda$  a proton and neutron have a potential energy of about  $25 \times 10^6$  ev. Thus  $g^2\lambda \simeq 25 \times 10^6$  ev. or

$$g \sim 5 \text{ to } 7 \text{ electron charges.} \quad \dots\dots(17)$$

So far the new theory is a classical theory. In the same way as the electromagnetic field, the new field has to be quantized. The quantization of the electromagnetic field leads to the existence of light quanta and in a similar way we shall obtain a new type of quanta arising from the quantized nuclear field  $\phi$ . While the light quanta arising from the electromagnetic field satisfy a wave equation of the type

$$\nabla^2\phi - \frac{1}{c^2} \frac{\partial^2\phi}{\partial t^2} = 0,$$

the wave equation for the Yukawa-potential will now be of the type

$$\nabla^2\phi + \lambda^2\phi - \frac{1}{c^2} \frac{\partial^2\phi}{\partial t^2} = 0. \quad \dots\dots(18)$$

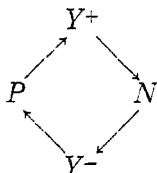
Equation (18) can be regarded as a wave equation for a particle with a rest mass

$$m = \frac{\lambda\hbar}{c} \simeq 150 \text{ electron masses.}$$

There is another very important feature of the new field  $\phi$ . From the saturation properties of the nuclear forces we know that the forces between a neutron and a proton are mainly exchange forces, i.e. they are connected with a steady permanent exchange of charge between the two particles. The proton gives its positive charge to the neutron becoming itself a neutron whilst the neutron turns into a proton. Forces of this type are very familiar in quantum physics, they are, for instance, responsible for the binding in a homoeopolar molecule. In order now to account for this exchange nature of the nuclear forces one has to attribute an ordinary electric charge to the new field  $\phi$ . The quanta arising from this quantized field will therefore also be charged, and it can be seen that they can have a positive or negative elementary charge  $e$ . Let us denote these charged quanta with a rest mass  $m \simeq 150$  electron masses by  $Y^+$ ,  $Y^-$  respectively. The production of the new field  $\phi$  by a proton or neutron can then also be considered as a virtual emission of such a  $Y$  particle by a proton or neutron as indicated by the equations (just as the electromagnetic field of an electron can be considered as a virtual emission of light quanta)

$$\left. \begin{aligned} N &\rightleftharpoons P + Y^-, \\ P &\rightleftharpoons N + Y^+. \end{aligned} \right\} \quad \dots\dots(19)$$

The charge is, of course, conserved in these processes. The forces between a proton and a neutron can then also be considered as due to a mutual emission and re-absorption of those particles,



An actual emission of such a particle can, of course, only take place if the necessary energy— $mc^2$  must be equal to at least 50 or 100 times  $10^6$  ev.—is available, and this is in general not the case for a nucleus. Only cosmic rays have sufficient energy to liberate those particles from a nucleus. If, for instance, a neutron or light quantum with an energy of at least  $100 \times 10^6$  ev. passes through a nucleus it might be expected that such a quantum would be emitted.

From the equations (19) we can deduce another important property of the quanta  $Y^\pm$ . Since both the proton and neutron have a spin  $\frac{1}{2}$  and obey the Pauli exclusion principle, it follows from general laws of quantum theory that the particles  $Y^+$ ,  $Y^-$  have an integral spin and obey Bose statistics just as is the case for light quanta but in contrast to electrons, protons and neutrons.

The quanta  $Y^+$ ,  $Y^-$  have exactly the properties which the heavy electrons occurring in cosmic radiation are now found to have. The mass calculated from the range of the nuclear forces is in excellent agreement with that measured on cosmic-ray tracks, table 2. It was in fact by the above arguments that Yukawa predicted the existence of heavy electrons three years before they were actually discovered.

There can be no doubt that the ideas developed above are in principle correct and that the heavy electrons found in cosmic rays are the quanta responsible for the nuclear field. However, in the form in which the new theory was originally published by Yukawa, it was not in agreement with all the experimental facts. It turned out, for instance, that the proton-neutron potential derived from the theory leads to a  $^1S$  state for the lowest state of the deuteron whilst the  $^3S$  state turned out to be repulsive. The experiments show that the ground state is a  $^3S$  state and that the  $^1S$  state, though still attractive, has a potential of only half of that of the  $^3S$  state. Thus the theory had to be modified in some way.

*Nuclear forces and magnetic moments.* After the existence of the heavy electrons was made probable by cosmic-ray experiments a consistent theory based on the general ideas discussed above has been developed independently by Fröhlich, Heitler and Kemmer<sup>(32, 44)</sup>, by Yukawa, Sakata and Taketani<sup>(67)</sup> and by Bhabha<sup>(11a)</sup>. The main point of the new theory is the following: The nuclear field is not described by a single potential  $\phi$  as in Yukawa's original paper but in a similar way to the electromagnetic field, by four potentials, a scalar potential  $\phi$  and a vector potential  $\mathbf{A}$ , from which six components of field strengths can be derived. The field equations are almost identical with Maxwell's equations, the only difference being that terms also occur which contain the universal length  $\lambda$  or the rest mass  $m$ . All four potentials satisfy in vacuo the wave equations (18) and an equation corresponding to the Lorentz condition

$$\operatorname{div} \mathbf{A} + \frac{1}{c} \frac{\partial \phi}{\partial t} = 0.$$

Apart from the terms  $\lambda^2 \phi$ ,  $\lambda^2 \mathbf{A}$  these equations are identical with Maxwell's equations. It turns out that heavy electrons can exist in different states of polarization, but, in contrast to light quanta, quanta with longitudinal polarization also exist.\* This can also be expressed in the following way: In the same sense as a light quantum, the heavy electron has a spin  $1\hbar$ .

In addition to the field equations there is an interaction of the heavy electron with the nuclear particles of such a form that the heavy electron can be emitted and absorbed by a proton and neutron, equation (19).†

There is another point in which Yukawa's original theory had to be extended in order to account for all the important experimental facts. From the scattering of two protons<sup>(62, 21)</sup> it is known that two protons have also an attractive potential of the same order of magnitude as a proton and neutron. The proton-neutron field

\* The existence of the latter is a consequence of the finite rest mass.

† This interaction is found to depend upon the spin direction of the proton (neutron) and contains two universal constants  $g$ ,  $f$ , both with the dimensions of a charge and of the order of 5 electron charges. They relate respectively to the longitudinal and transverse heavy electrons.

cannot, of course, be of an exchange type because no exchange of charge can take place between two particles with the same charge. The quantization of the proton-proton field leads, therefore, to quanta which carry no charge but have the same rest mass  $m$ . We call these neutral particles "neutrettos". The theory has, therefore, to be built up in such a way that it includes also neutral particles. This can be done in an exceedingly simple way as is shown by Kemmer<sup>(45)</sup>. We shall not discuss the details of this theory here but confine ourselves to a discussion of the main results.

The main results of this theory are the following:

- (a) The lowest state of the deuteron is a  $^3S$  state which is always attractive.
- (b) The  $^1S$  state of the deuteron is higher than the  $^3S$  state and the constants occurring in the theory can be chosen so that its position is in agreement with experiment.
- (c) The proton-proton potential is exactly equal to the proton-neutron potential in the  $^1S$  state.
- (d) Taking the mass  $m$  of the heavy electron from cosmic rays all forces have the range  $1/\lambda \simeq 3 \times 10^{-13}$  cm. in agreement with experiment. For  $S$  states the proton-neutron and proton-proton potentials have the form  $\text{const.} \times e^{-\lambda r}/r$ .
- (e) The theory leads to an anomalous magnetic moment of the proton  $\mu_P$  and neutron  $\mu_N$  which are of the order of magnitude actually observed and have the right sign (positive for the proton, negative for the neutron).
- (f) The ratio of the anomalous magnetic moments\* depends only upon the mass of the heavy electron (and of the proton) and can therefore be calculated exactly. It is in good agreement with the experiments<sup>(31)</sup>.†

The existence of an anomalous magnetic moment can be understood in the following way. A free proton is surrounded by a nuclear field or, in other words, is capable of emitting a positive heavy electron transforming itself into a neutron. A free proton can, of course, not actually emit a heavy electron (because the energy is not available) but it will spend a certain fraction of time in a neutron state in which it has virtually emitted a heavy electron. Consider now a free proton with its spin directed upwards. Then the theory shows that it will spend a certain fraction of time as a neutron and a heavy electron with an angular momentum  $+\hbar$  about the axis of spin direction. Because of the conservation of angular momentum the neutron has then a spin directed downwards. In this state the heavy electron will have a large magnetic moment—about 12 times larger than a Bohr nuclear magneton—because its mass is about 12 times smaller than that of the proton. The fraction of time which the proton spends in this dissociated state is calculable from the

\* Subtracting from the magnetic moment of the proton the normal Dirac-magneton which is equal to one Bohr nuclear magneton  $\mu_0$ .

† The theory gives the formula

$$\frac{\mu_P - \mu_0}{-\mu_N} = \frac{M}{M+m} - \frac{m}{M},$$

where  $M$  is the mass of the proton and  $m$  the mass of the heavy electron.

theory and is of the order  $g^2/\hbar c$ . The additional magnetic moment of the proton is, therefore, in units of the Bohr nuclear magneton  $\mu_0$

$$\frac{\mu_P - \mu_0}{\mu_0} \simeq \frac{g^2}{\hbar c} \frac{M}{m} \simeq 2, \quad \dots\dots(20)$$

which is just of the right order of magnitude. The same consideration applies to the neutron, the sign of  $\mu_N$  is the negative one because a neutron emits a negative heavy electron.

Although there can be little doubt that this theory is correct in its outline, it must, of course, only be regarded as a preliminary theory. In particular it has turned out that it cannot be applied for very fast heavy electrons with velocity nearly equal to that of light.\* This point will be important for the application to cosmic rays.

*Heavy electron and cosmic rays.* Very little can be said yet about what happens when a heavy electron passes through matter. The mass of the heavy electron is so large that the energy loss by radiation (which is proportional to  $1/m^2$ ) is negligible. On the other hand, the measurements of the energy loss (figure 7, § 6) indicate that the energy loss is still very much larger than that by ionization only. There must be, therefore, some other processes by which a heavy electron can lose energy. The experiments discussed in § 2 show that a heavy electron is also capable of producing showers and bursts. If we consider the part of the Rossi curve, figure 4, which is due to showers produced by the penetrating rays, it seems that there are two types of showers which the latter can produce. The curve (b) rises within the first few cm. of lead very steeply and remains then practically constant. This indicates a type of shower consisting of particles with a range of only a few cm. of lead. These particles are certainly electrons with an energy of not much more than  $10^8$  ev. Thus we expect a heavy electron to be capable of producing showers consisting of electrons of comparatively small energy. At a thickness of 15 to 20 cm. of lead the shower curve rises again and has a second maximum. This shows that some of the shower particles have themselves a penetrating power up to 20 cm. of lead and can, therefore, only be heavy electrons themselves. Thus a heavy electron is also capable of creating showers consisting of heavy electrons.

We might also mention that several authors have come to the conclusion that a part of all these showers is due to a non-ionizing (i.e. neutral) radiation with a penetrating power similar to, or even larger than, that of the heavy electrons<sup>(7, 48)</sup>.

It seems that the heavy electron is far from being a stable particle and disintegrates in some way as soon as it reaches an energy less than  $2 \times 10^8$  ev., because practically no heavy electrons are observed below this energy.† This would lead to the conclusion that the heavy electrons cannot themselves come from outside the earth (their life-time would not be long enough to travel astronomical distances),

\* For the reasons for the breakdown of the theory at high energies the reader is referred to the original papers<sup>(32, 44, 67)</sup>.

† It has been suggested by Yukawa<sup>(65)</sup> and also by Bhabha<sup>(11)</sup> that the heavy electron can disintegrate spontaneously, without the presence of other particles, into an ordinary electron and neutrino ( $\beta$  decay of the heavy electron).

but are created as secondaries by the primary particles, e.g. by electrons, in the atmosphere. The same conclusion has also been reached by several authors<sup>(20)</sup> from indirect experimental evidence concerning the energy balance in the atmosphere. Although no direct proof of this fact is available yet, we think it very probable that the heavy electrons are secondaries and that the electrons with the primary energy spectrum described in § 5 are the only really primary particles coming from the universe.

We shall not discuss in this report all the different opinions and views which have been expressed about the behaviour of the heavy electrons in cosmic radiation so far; they will certainly be subjected to considerable alterations in the near future.\* But one question re-presents itself in connexion with the theory of the heavy electron, namely: is the theory, which has been successful in the explanation of the nuclear properties, capable of accounting for the behaviour of the heavy electrons in cosmic radiation, and what are its consequences?

Unfortunately, the theory developed in the earlier part of this section is only valid for small energies and breaks down for cosmic-ray energies. In spite of this fact, it might be hoped that the theory would give at least some qualitative information about the processes to be expected, and since it exhibits some features which are not common to any other of the existing theories, we shall briefly mention some of its consequences.

The main assumption made in this theory is expressed in the equations (19) stating that in suitable circumstances a heavy electron can be absorbed or emitted by a nuclear particle. If such an absorption takes place the energy of the primary heavy electron must be given up in some way. This leads to a number of different processes which could be expected to occur if a heavy electron passed through a nucleus<sup>(40)</sup>:

(a) A heavy electron,  $Y^-$  say, is absorbed by a proton, and re-emitted into another direction. This process represents simply an anomalous scattering by a nucleus. A certain amount of energy is given up to the nucleus (in order to conserve momentum) and the primary electron loses a certain amount of energy. Possibly this or similar processes are responsible for the additional energy loss of heavy electrons.

(b) A heavy electron is absorbed by a nuclear particle, the energy being given up in the form of a light quantum

$$Y^- + P \rightarrow N + \hbar\nu. \quad \dots\dots(21)$$

Subsequently, the light quantum will create an ordinary cascade shower. Thus an electron shower is produced, as was in fact seen to be the case from the shape of the Rossi curve.

Electron showers can also be produced if a heavy electron hits an electron in an atomic shell in a direct head-on collision, transferring a large amount of energy to the latter. The fast electron will then produce a cascade shower. The latter type of process leads, however, only to comparatively small showers<sup>(20)</sup>.

\* Half-empirical attempts to bring the observations into a consistent scheme have been made by Nordheim<sup>(53)</sup> and Euler<sup>(30)</sup>.

(c) A very surprising and perhaps the most important feature of the theory of the heavy electron is the following. If a heavy electron of sufficiently high energy, e.g. larger than about 3 to 5 times  $10^8$  ev., interacts with a nucleus, a so-called multiple process or explosion may occur. It may then be that a large number of heavy electrons is emitted simultaneously in one elementary act, for instance:

$$Y^- + P \rightarrow P + Y^- + Y^+ + Y^-, \quad \dots\dots(22)$$

The number of heavy electrons emitted is only restricted by the initial energy of  $Y^-$ . The existence of those explosive processes was first postulated by Heisenberg<sup>(35)</sup> on the grounds of Fermi's theory of  $\beta$  decay and made responsible for the majority of big showers and bursts. It has since been shown by Heitler<sup>(40)</sup> that the theory of the heavy electron leads to the same qualitative consequence and we believe now that it is the type of process given by equation (22) which actually is responsible for the explosions occurring in nature.

The process (22) gives rise to the production of a shower consisting of heavy electrons, i.e. of penetrating particles only. Penetrating showers really occur in nature, we have seen that they are responsible for the second maximum of the Rossi curve. In particular a discussion of the transition curve for bursts by Euler<sup>(30)</sup> has shown that a large fraction of the bursts must be ascribed to multiple processes of the type indicated by equation (22).

(d) So far we have only considered processes for which the nuclear particles can be considered as free. In a heavy nucleus this is not quite the case. In all the processes (a), (b), (c) a certain amount of energy is transferred to the nuclear particle. According to Bohr's theory the latter will interact at once with other nuclear particles which results in a general excitation of the whole nucleus. It might then happen that the nucleus becomes very highly ( $10^8$  ev. and more) excited. In course of time the nucleus will release a number of its constituents, emitting subsequently a few protons and neutrons in all directions until its energy has decreased to the ground state. Thus, we shall find a shower containing heavy particles. Those nuclear explosions have occasionally been observed, we have discussed them in § 2. They can, of course, also be produced by a direct head-on collision of any very energetic cosmic-ray particle with a nucleus<sup>(36)</sup>.

(e) Finally, we might mention that we should also expect neutral penetrating particles in cosmic rays. From the discussion of the proton-proton forces it has been seen to be probable that these neutral particles with mass  $m \simeq 150$  electron masses also exist. We have called them "neutrettos". In cosmic rays they give rise to almost the same processes as the charged heavy electrons—except of course those for which the charge is essential. In particular, it can happen that a heavy electron is transformed into such a neutretto (denoted by  $Y^0$ ) during a collision with a nuclear particle, for instance:

$$Y^- + P \rightarrow N + Y^0, \quad \dots\dots(23)$$

and vice versa. In this way neutrettos can be emitted and absorbed. They give rise also to all the shower processes of the types (c) and (d), but not of the type (b) because for the emission of light the charge is essential. It seems that there are

some indications in cosmic rays for those showers produced by neutral particles<sup>(4)</sup>, but it would lead too far to discuss them in detail here.

(f) On the basis of this theory it can also be understood that the heavy electrons are secondaries created by ordinary electrons and light quanta. The reverse process of (b) for instance, gives rise to a creation of a heavy electron by a light quantum in a collision with a nucleus, and a rough estimate shows that the cross-section is large enough to account for the actual numbers of heavy electrons created in the high atmosphere.

On the whole our knowledge about the hard cosmic radiation is still very poor, both from the experimental and theoretical point of view. On the other hand, the situation has been clarified sufficiently to reveal the intimate connexion between these particles and the nuclear properties. It is clear that future experiments on the hard cosmic rays could give substantial contributions to a further deepening of our knowledge about the fundamental properties of the elementary particles.

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